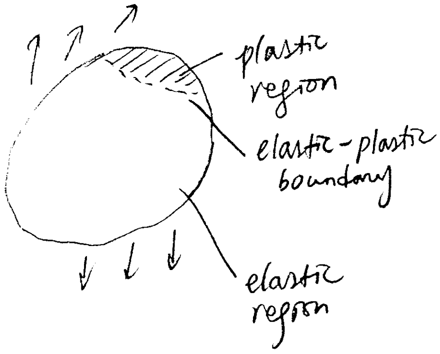
Theory and Applications of Inelasticity

In this course, we discuss plasticity theory, its applications, and fracture.

Study of (ductile) fracture in engineering materials requires an understanding of plasticity in front of the crack tip.

This course is the sequel of ME340 Elasticity, in which we limit the load such that the stress is below yield stress everywhere.

In this course, we allow the load to reach (or exceed) yield stress and allow the structure to develop (permanent) plastic strain.



The goal is (still) to find the stress, strain, and displacement fields.

Usually, only a part of the structure will undergo plastic deformation, while the rest remains in the elastic regime.

So as an intermediate step, we also need to determine the elastic-plastic boundary.

The other complication is that, in the plastic region, the total strain is the sum of elastic and plastic strain.

We shall assume that the elastic strain is proportional to stress. But the plastic strain and total strain are in general not proportional to stress. The plastic strain and total strain depend on the history of loading.

Goal of this course:

1. The theory of plasticity
   i.e. the physics behind the phenomena and how to describe it mathematically (PDE)
2. Method of solution (not including FEM)
   Analytical method
   Semi-analytical method involving Matlab
3. Solution of classical plasticity problems

A major goal of this course is to develop an 'intuitive feel' of plasticity problems, so that

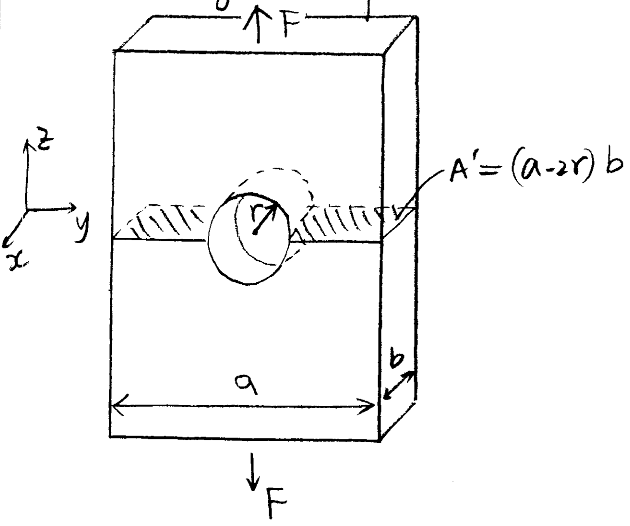
- when you encounter a simple plasticity problem in your research, you can use an analytical solution, or write a simple numerical code to solve it.
- when you encounter an finite element method (FEM) solution, you can quickly tell whether the solution 'makes sense'.

Relationship with other courses

- ME340 "Elasticity" (or equivalent as prerequisite)
- CEE292 "Computational Micromechanics" (Spring 2013)
  deals with the same physical phenomenon — plasticity
  focuses on numerical algorithm

This course focuses on the physics of the solution, analytical method.

Motivating Examples



From Elasticity theory, we know

$$(\sigma_{zz})_{max} = K \cdot \frac{F}{A'}$$

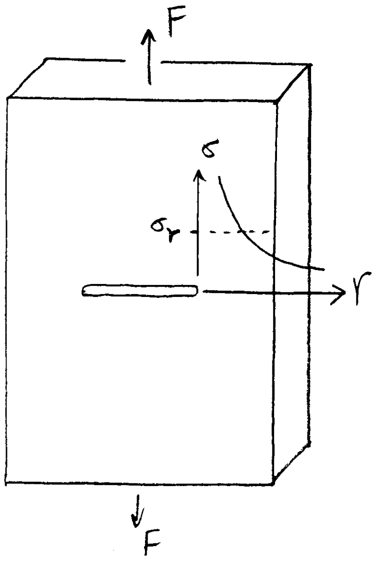
$\uparrow$ Stress concentration factor

when stress is below σ_Y (yield stress) every where.

$K = 3$ when $a \gg r$.

Q: what is the stress distribution when $(\sigma_{zz})_{max} \geqslant \sigma_Y$?

Q: what is the size of the plastic zone?



Slit like crack

Elasticity theory predicts

$$\sigma \sim \frac{1}{\sqrt{r}}$$

which surely will exceed σ_Y for arbitrarily small F.

Q: How is stress field modified when σ_Y is accounted for ?

Q: what is the size (shape) of the plastic zone ?

Q: what is the effect of σ_Y on fracture toughness ?