ME340B – Elasticity of Microscopic Structures – Wei Cai – Stanford University – Winter 2004

Problem Set 6. Cracks

Due date: Mar 2, 2005

Problem 6.1 (15') Plane strain and plain stress equivalence.
Let the elastic stiffness tensor of a homogeneous solid be C_{ijkl} and its inverse (compliance tensor) be S_{ijkl}. In the plane strain problem, $e_{13} = e_{23} = e_{33} = 0$. Let the 2-dimensional elastic stiffness tensor be c_{ijkl}, i.e.,

$$\sigma_{ij} = c_{ijkl} e_{kl} \quad \text{for } i,j,k,l, = 1,2 \ \ \text{(plane strain)} \tag{1}$$

Obviously, $c_{ijkl} = C_{ijkl}$ for $i,j,k,l = 1,2$.

For a plain stress problem, $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$. Let the 2-dimensional elastic compliance tensor be $\tilde{s}_{ijkl}$, i.e.,

$$e_{ij} = \tilde{s}_{ijkl} \sigma_{kl} \quad \text{for } i,j,k,l, = 1,2 \tag{2}$$

Obviously, $\tilde{s}_{ijkl} = S_{ijkl}$ for $i,j,k,l = 1,2$. The inverse of $\tilde{s}_{ijkl}$ (in 2-dimension) is the effective elastic stiffness tensor in plain stress, $\tilde{c}_{ijkl}$.

(a) For isotropic elasticity, write down the explicit expression for c_{ijkl} and $\tilde{c}_{ijkl}$.

(b) The Kolosov's constant is defined as

$$\kappa = \begin{cases} 3 - 4\nu & \text{for plane strain} \\ \frac{3-\nu}{1+\nu} & \text{for plane stress} \end{cases}$$

Express c_{ijkl} and $\tilde{c}_{ijkl}$ in terms of μ and κ. (They should have the same expression now.)

Problem 6.2 (15') Mode II crack
(a) Derive the eigenstrain of equivalent inclusion for a slit-like crack (width $2a$) under uniform shear σ_{12}^A in plane strain.

(b) Derive the stress distribution in front of the crack tip. What is the stress intensity factor $K_{II} = \lim_{r \to 0} \sigma_{12}(r) \sqrt{2\pi r}$, where $r = x - a$ is the distance from the crack tip?