Problem Set 5. Eshelby's Inclusion

Due date: Feb 9, 2005

Problem 5.1 (15') Use work method to derive the energy inside the inclusion E^I and inside the matrix E^M for an ellipsoidal inclusion in an infinite matrix. Follow the Eshelby's 4 steps to construct the inclusion.

(a) What are the forces applied to the inclusion and to the matrix in all 4 steps?

(b) What are the work done to the inclusion and to the matrix in all 4 steps?

(c) What is the elastic energy inside the inclusion E^I, and what is the elastic energy inside the matrix E^M at the end of step 4?

Problem 5.2 (15') Spherical inclusion. The Eshelby's tensor of a spherical inclusion inside an infinite medium is (see Lecture Note 2),

$$\mathcal{S}_{ijkl} = \frac{5\nu - 1}{15(1-\nu)}\delta_{ij}\delta_{kl} + \frac{4-5\nu}{15(1-\nu)}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{1}$$

Consider a spherical inclusion of radius R with a pure shear eigenstrain $e^*_{12} = \varepsilon$ (other components of $e^*_{ij} = 0$).

(a) What is the total elastic energy of the system E as a function of R?

(b) Now apply a uniform stress field $\sigma^A_{12} = \tau$ to the solid (other stress components are zero). What is the total elastic energy $E(R)$?

(c) What is the enthalpy of the system $H(R)$? What is the driving force for inclusion growth, i.e. $f(R) = -dH(R)/dR$?

[Hint: Consider the solid has a finite but very large volume V. The external stress is applied at the external surface. Volume V is so large that the Eshelby's solution in infinite solid remains valid.]