ME340B – Elasticity of Microscopic Structures – Wei Cai – Stanford University – Winter 2004

# Problem Set 1. Index Notation and Gauss's Theorem

Due date: Jan 12, 2005

**Problem 1.1** (10') Index notation.

(a) Show that $\epsilon_{mkq}\epsilon_{nkq} = 2\delta_{mn}$.

(b) Consider a rank-two tensor $p_{ij} = a\delta_{ij} + bz_iz_j$, where $\mathbf{z}$ is a unit vector ($z_iz_i = 1$). Find the inverse $q_{ij}$ of $p_{ij}$, which is defined through $q_{ij}p_{jk} = \delta_{ik}$.
[ Hint: suppose $q_{ij}$ also has the form of $q_{ij} = c\delta_{ij} + dz_iz_j$. ]

**Problem 1.2** (10') Tensor symmetry.

Any second rank tensor $A_{ij}$ can be decomposed into its symmetric and antisymmetric parts

$$A_{ij} = A_{(ij)} + A_{[ij]}$$

where

$$A_{(ij)} = \frac{1}{2}\left(A_{ij} + A_{ji}\right)$$

is the symmetric part and

$$A_{[ij]} = \frac{1}{2}\left(A_{ij} - A_{ji}\right)$$

is the antisymmetric part.

(a) Show that if $A_{ij}$ is a symmetric tensor, and $B_{ij}$ is an arbitrary tensor, then,

$$A_{ij}B_{ij} = A_{ij}B_{(ij)} \qquad (1)$$

(b) Show that if $A_{ij}$ is an antisymmetric tensor, then

$$A_{ij}a_ia_j = 0$$

**Problem 1.3** (10') Gauss's Theorem.

(a) For a elastic body $V$ with surface $S$ in equilibrium under surface traction $T_i$ and zero body force ($b_i = 0$), show that

$$\int_S T_i u_i \mathrm{d}S = \int_V \sigma_{ij} e_{ij} \mathrm{d}V$$

where $u_i$, $\sigma_{ij}$, $e_{ij}$ are displacement, stress and strain fields.
[ Hint: Use the result in Problem 1.2. ]

(b) Show that the average stress in the elastic body under zero body force is,

$$\overline{\sigma}_{ij} = \frac{1}{2V} \int_S (T_i\, x_j + T_j\, x_i) \mathrm{d}S$$