Problem 1.1 (10’) Index notation.

(a) Show that $\epsilon_{mkq}\epsilon_{nkq} = 2\delta_{mn}$.

(b) Consider a rank-two tensor $p_{ij} = a\delta_{ij} + bz_iz_j$, where $z$ is a unit vector ($z_iz_i = 1$). Find the inverse $q_{ij}$ of $p_{ij}$, which is defined through $q_{ij}p_{jk} = \delta_{ik}$.

[Hint: suppose $q_{ij}$ also has the form of $q_{ij} = c\delta_{ij} + dz_iz_j$.]

Problem 1.2 (10’) Tensor symmetry.

Any second rank tensor $A_{ij}$ can be decomposed into its symmetric and antisymmetric parts

$$A_{ij} = A_{(ij)} + A_{[ij]}$$

where

$$A_{(ij)} = \frac{1}{2} (A_{ij} + A_{ji})$$

is the symmetric part and

$$A_{[ij]} = \frac{1}{2} (A_{ij} - A_{ji})$$

is the antisymmetric part.

(a) Show that if $A_{ij}$ is a symmetric tensor, and $B_{ij}$ is an arbitrary tensor, then,

$$A_{ij}B_{ij} = A_{ij}B_{(ij)} \hspace{1cm} (1)$$

(b) Show that if $A_{ij}$ is an antisymmetric tensor, then

$$A_{ij}a_ia_j = 0$$
Problem 1.3 (10’) Gauss’s Theorem.

(a) For a elastic body $V$ with surface $S$ in equilibrium under surface traction $T_i$ and zero body force ($b_i = 0$), show that

$$\int_S T_i u_i dS = \int_V \sigma_{ij} e_{ij} dV$$

where $u_i$, $\sigma_{ij}$, $e_{ij}$ are displacement, stress and strain fields.

[ Hint: Use the result in Problem 1.2. ]

(b) Show that the average stress in the elastic body under zero body force is,

$$\overline{\sigma}_{ij} = \frac{1}{2V} \int_S (T_i x_j + T_j x_i) dS$$