

Final Exam

Issued: Mar. 9, 2005

Due: Mar. 16, 2005

Problem F.1 (20') Orowan's equation

$\dot{\epsilon} = \rho b v$ relates the plastic strain rate $\dot{\epsilon}$ to the mobile dislocation density ρ , Burgers vector b and average dislocation velocity. To prove this equation (in a simple case), consider a planar closed dislocation loop in a finite, otherwise homogeneous linear elastic solid under zero tractions at the external surface. Let the area of the loop change by ΔA . As a result, the average strain of the body change by Δe_{ij} . Let the n_i be the normal vector of the plane.

Use Betti's theorem to show that

$$\Delta e_{ij} = \frac{1}{2V} (n_i b_j + n_j b_i) \Delta A \quad (1)$$

[Hint: model the dislocation loop with an equivalent inclusion.]

Problem F.2 (20') J-integral and Peach-Koehler force

Consider an infinite straight screw dislocation along z -axis with Burgers vector b_z in an infinite isotropic medium under uniform applied stress σ_{yz}^A .

- (a) What is the driving force (per unit length) on this dislocation by Peach-Koehler formula?
- (b) Compute the J-integral around this dislocation in both x and y directions, i.e., J_x and J_y , based on the known elastic field of the screw dislocation (Hirth and Lothe 1982). How do they compare with the Peach-Koehler force? Print results for both terms in the J-integral. Notice that in this case the J-integral is evaluated on a closed surface surrounding the dislocation.

Problem F.3 (40') Crack nucleation

- (a) Consider cutting an isotropic solid into two halves by a mathematical plane and uniformly separating the two halves normal to the plane by u . Let the traction force $f(u)$ per unit area across the plane take the following form (universal binding curve),

$$f(u) = A u e^{-\alpha u} \quad (2)$$

What is the surface energy per unit area $\gamma(u)$?

(b) Put the solid under plane strain condition. For a slit like crack with length $2a$ along x -axis under uniform applied stress σ_{yy}^A , what is the critical condition that the crack will grow according to Griffith's criteria? [Hint: you only need to figure out what γ value to use in the Griffith's criteria.]

(c) Suppose the stress distribution $\sigma_{yy}^A(x)$ in the absence of the crack on the x -axis is not uniform. Let $u(x)$ be the separation of the upper and lower halves of the solid across x -axis. Then the Gibbs free energy of the system can be written as,

$$\Delta G = -K \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x)\rho(x') \ln |x - x'| dx dx' + \int_{-\infty}^{\infty} \gamma(u(x)) dx - \int_{-\infty}^{\infty} \sigma_{yy}^A(x) u(x) dx \quad (3)$$

where $\rho(x) = du(x)/dx$, $K = \mu/(4\pi(1-\nu))$. The first term describes the elastic interactions in the absence of applied stress. The second term describes the surface energy and the third term describes the work done by the applied stress. The function $u(x)$ that minimizes ΔG corresponds to the physical separation of the planes under applied stress $\sigma_{yy}^A(x)$.

Write a Matlab program that can solve for $u(x)$ that minimizes ΔG for a given $\sigma_{yy}^A(x)$. Represent $u(x)$ by a piece-wise linear function on a uniform grid with size d : $u_i = u(id)$, $i = -N, -N+1, \dots, N$. Apply boundary conditions: $\rho(x) = 0$, $x < -Nd$ or $x > Nd$. Express ΔG as a function of u_i . Starting with initial condition: $u_i = 0$. Compute derivative $g_i = \frac{\partial \Delta G}{\partial u_i}$. Update u_i by $u_i - g_i \cdot \delta$ with a small step size δ . Repeat the iteration until convergence has been reached.

Let

$$\sigma_{yy}^A(x) = B e^{-\beta x^2} + C \quad (4)$$

$\mu = 1$, $\nu = 0.3$, $A = 0.1$, $\alpha = 1$, $B = 0.05$, $\beta = 0.01$, $C = 0$, $d = 1$, $N = 500$, $\delta = 0.1$.

Plot the resulting $u(x)$.

(d) Increase the background stress C in step of 0.01 while keeping other parameters fixed. What is the critical value of C where the system is unstable against crack nucleation and propagation?

[Hint:

$$\int_0^d \int_{nd}^{(n+1)d} \ln |x - x'| dx dx' = \left[\ln d - \frac{3}{2} + f(n) \right] d^2 \quad (5)$$

where

$$f(n) = \frac{(n+1)^2}{2} \ln(n+1) - n^2 \ln n + \frac{(n-1)^2}{2} \ln(n-1) \quad (6)$$

for $n > 1$ and $f(1) = \ln 4$, $f(0) = 0$.]