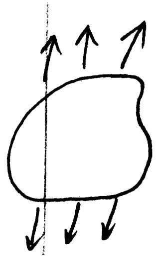
Elasticity Theory



determines stress and displacement in a body as a result of applied (mechanical or thermal) load

— Barber

A body under elastic deformation reverts to its original state on the removal of loads.

* Elasticity theory is also useful for inelastic deformation, such as fracture and plasticity, by studying the microscopic agents of inelasticity, such as a crack or a dislocation

— this is called micromechanics

In this course, we will focus on linear, infinitesimal elasticity: stress and displacements are linear with loads.

Linear superposition can be used to construct solutions.

(* Statics only)

Compared with the undergraduate "Mechanics of Materials" course (ME80 in Stanford), which makes plausible but unsubstantiated assumptions.

Elasticity Theory:
- is a more rigorous treatment
- only makes mathematical assumptions (usually in the last step, to help solve the equation) instead of physical assumptions (hard to justify)
- allows us to assess the quality of assumptions made in "mechanics of materials".
- uses more advanced mathematical tools tensors, partial differential equations, Fourier transform ...

Outline of this course

1. fundamental variables and equations of elasticity:

   variables: stress, strain, displacement, elastic constants

   equations: equilibrium, compatibility

2. Methods to solve these equations:

   2D problems: stress functions

   3D problems: Green functions

3. Applications

   - are going to be interwined with the methods
   - Matlab will be used both for numerical and for symbolic calculations.

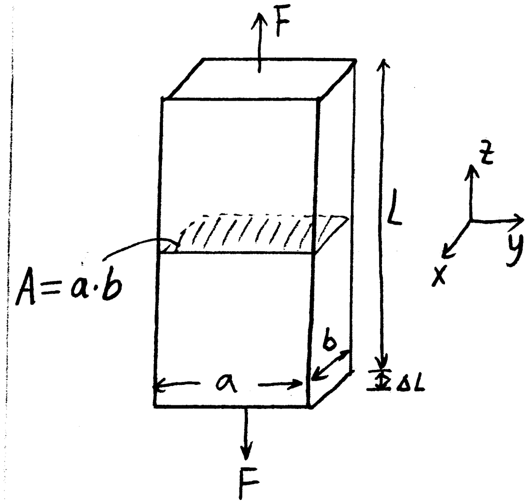
Relationship with other courses.

- undergraduate "Mechanics of materials" as pre-requisite
- students already taken "continuum mechanics" will benefit from the similar notation and assumptions. But "continuum mechanics" is not a prerequisite for this class, because we will make a lot of simplifications (linear, infinetesimal elasticity) so that we don't need the full-blown continuum mechanics (finite deformation, etc.)
- Many elasticity problems today are solved by numerical methods such as the "Finite Element Method"

  Elasticity theory provides the fundamental equation to be solved by the numerical methods.

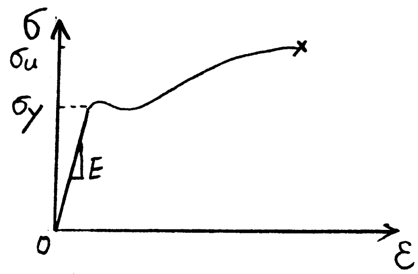
  Analytic solutions also reveal the physics that are difficult to see by numerical methods; they also motivate the

Example 1. (Mechanics of Materials)



(axial) stress: $\sigma_{zz} = \frac{F}{A}$

(axial) strain: $\varepsilon_{zz} = \frac{\Delta L}{L}$



σ_y: yield stress

E: Young's modulus

σ_u: ultimate strength

Mechanics of materials:

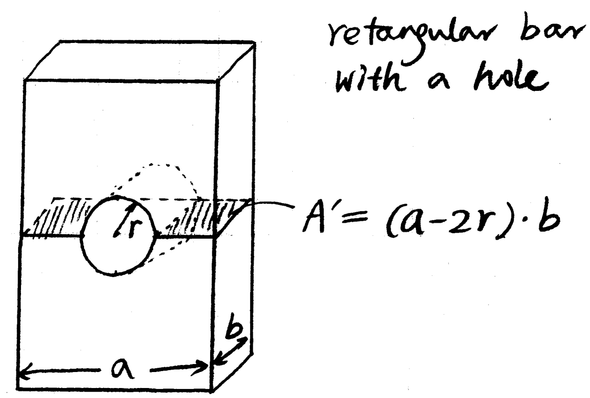
if $\sigma_{zz} < \sigma_p$ (proportional limit, $\sim \sigma_y$)

then $\sigma_{zz} = E \cdot \varepsilon_{zz}$, $\Delta L = \frac{FL}{EA}$

$\sigma_{zz} < \sigma_y$ is required to prevent yielding

$\sigma_{zz} < \sigma_u$ is required to prevent fracture.

retangular bar with a hole



$(\sigma_{zz})_{max} = K \cdot \frac{F}{A'}$

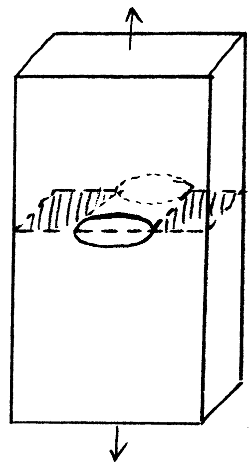
stress concentration factor

Q: where does it come from?

- in "Mechanics of Materials" there is a look-up table.
- in "Elasticity", we compute the stress distribution around the hole.

We will show analytically $K = 3$ when $a \gg r$

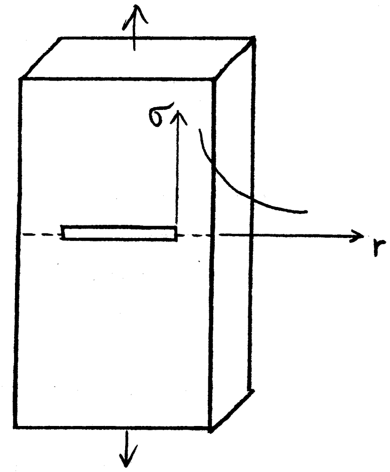
* Notice that K is independent of the size (r) of the hole by itself.
It only depends on the ratio $\frac{r}{a}$.
This "scale invariance" is an important feature of Elasticity theory.



elliptic hole

Q: what is the stress concentration factor?

Elasticity theory can answer!



Slit like crack

Stress field becomes singular at crack tip.

Elasticity theory predicts

- $\sigma \sim \frac{1}{\sqrt{r}}$
- stability criteria for crack (advancement)