The Deformation of a Tilt Boundary under Applied Forces

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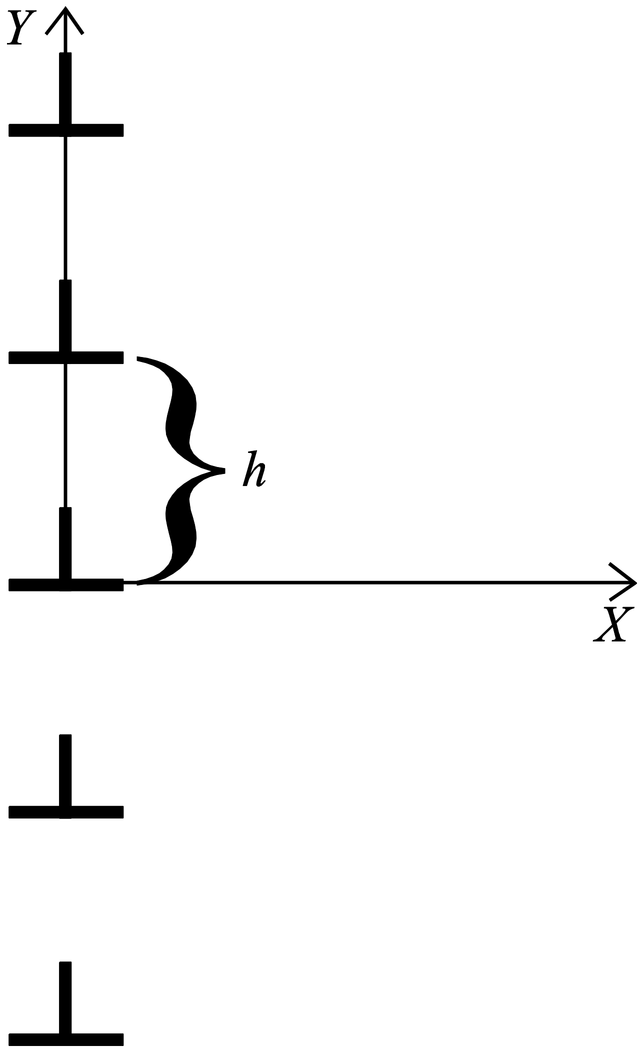
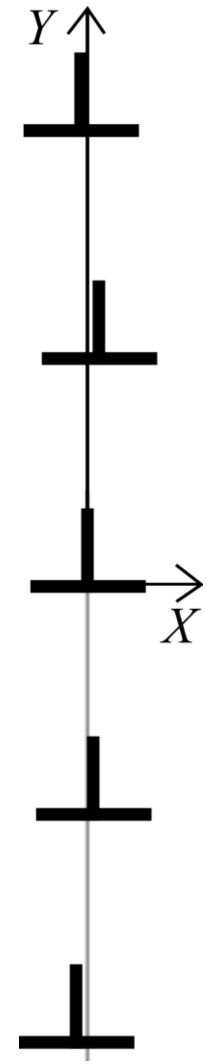
Previous Efforts

- Vreeland had already "considered the equilibrium, under an applied stress, of a tilt boundary in which some of the dislocations have become pinned," but with two limitations:
  - Dislocation configuration is periodic
  - Forces on the dislocations are expressed as functions of their positions, requiring solving simultaneous equations.

Periodic Deformation



Originally a dislocation wall with spacing h. When a force is applied, the dislocations will move along the x-axis slightly, resulting in the right configuration. Initially, we will limit these displacements to ensure that the line joining any two dislocations makes a small angle with the y-axis.



Equilibrium

- In general, we can write: $\sigma_{xy} \approx -D\frac{x}{y^2}$, neglecting higher order terms in x.
- When the n^{th} dislocation in the boundary is displaced a distance x_n, there will be a force on it, due to the m^{th} dislocation.
- If there is also an applied force F_n, the condition for equilibrium is:

$$F_n = \frac{Db}{h^2}\sum_{l=1}^{\infty}\frac{(x_n - x_{n+l}) + (x_n - x_{n-l})}{l^2} \qquad \text{if } |x_m - x_{m-1}| \ll h$$

General Periodic Displacement

- We can write, for a period of N dislocations:

$$x_n = \sum_{p=0}^{N-1} a_p e^{\frac{2\pi i p n}{N}}$$

- Two unknowns: F_n and a_n.
- We end up with two conditions:

$\sum F_n = 0$ and $x_n = \frac{h^2}{\pi^2 D b} \sum_{l=0}^{N-1} C_l F_{n+l}$ where $C_l = \sum_{p=1}^{N-1} \frac{\cos\left(\frac{2\pi l p}{N}\right)}{p}$

- Note: for $F_n = F + F_n'$ where F is the same for all dislocations, F gives no contribution to displacements.

Non-Periodic Deformations

- General procedure is to (carefully) allow N to tend to infinity.

$$x_n = -\frac{h^2}{\pi^2 Db}\sum_{l\neq 0} F_{n+l}(C + \log|l| + \delta_l)$$

where C = 2.4151 and $\delta_l = \int_{2\pi l}^{\infty} \frac{\cos(t)}{\mathrm{t}}\mathrm{dt} = \frac{1!}{(2\pi l)^2} - \frac{3!}{(2\pi l)^4} + \frac{5!}{(2\pi l)^6} - \cdots$

- Note: the further we go from the point of application of the force, the greater the displacement it produces (due to log term).
  - This is related to the fact that the forces no longer form an equilibrium system.

Large Displacements

- We have used $\frac{-bDx}{y^2}$ to calculate the force between two dislocations, and it is valid only when the displacements are sufficiently small.
- We can use a relaxation technique to calculate the actual forces.
  - First, we use our formula, to get displacements based on forces. However, the force on the dislocations at these displacements will not in general be zero.
  - So we take the largest residual and subtract it off and recalculate displacements until the error is below the tolerance
- This is worked out for a pinned dislocation as an example.