Dislocation Group Dynamics
I. Similarity Solutions of the n-body Problem

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General Overview

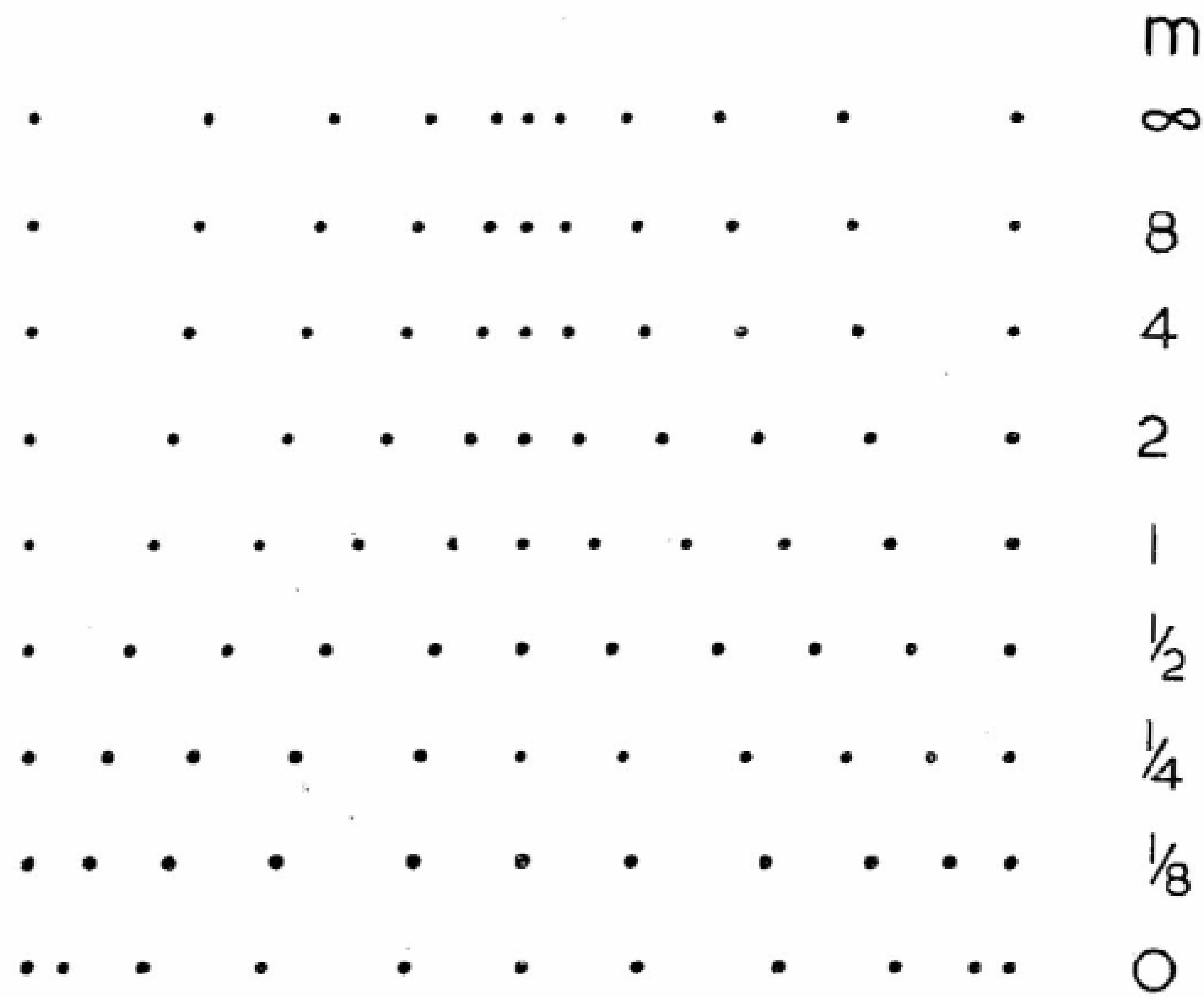
- Similarity motion is explored for many situations:
  - Identical dislocations with no applied field
  - Identical dislocations with a locked dislocation
  - Identical dislocations with an applied stress field
  - Identical dislocations with a time-dependent applied stress field
  - Identical dislocations with different (constant) mobilities
  - Identical dislocations with position-dependent mobilities
- A few other situations are glanced at in passing

Background Equations and No Applied Stress

- Velocity: $$\frac{dx_j}{dt} = M\{\sigma(x_j)\}^m, \qquad j = 1, \ldots, n$$
- Stress: $$\sigma(x_j) = S(x_j) + A\sum_{\substack{i=1 \\ \neq j}}^{n} \frac{1}{x_j - x_i}, \quad j = 1, \ldots, n$$
- M = mobility
- S = applied Stress, A = constant
- No applied stress; we want this form: $x_j = g(t)X_j$
- Result is separable: $$g\left(\frac{dg}{dt}\right)^{1/m} = X_j^{-1/m}\sum{}' \frac{1}{X_j - X_i}, \quad j = 1, \ldots, n$$
- This can be solved to give: $$g(t) = \{(m+1)t\}^{1/(m+1)}$$
  - When m = 1, the solutions are the roots of the n^{th} Hermite polynomial.

Similarity arrangements of 11 dislocations during free expansion

Note: These arrangements have been scaled to occupy the same overall length.



Variations

- Locked Dislocation
  - Same as having an applied field of $S(x) = \frac{1}{x}$
- Applied Field of the Form: $S(x) = cx^{1/m}$
  - Solving, the time scale is $t = \int \frac{dg}{(g^{-1} + cg^{1/m})^m}$
  - This can be solved explicitly for rational m, but only inverted if m=1. The motion then has three different forms.
  - c>0 (accelerating expansion) and t > 0:

$$g = \sqrt{\frac{e^{2ct} - 1}{c}}$$

Variations

- c<0 (equilibrium) of two types (set c' = -c):
  - Expansion toward equilibrium, t>0: $g = \sqrt{\frac{1 - e^{-2c't}}{c'}}$
  - Contraction toward equilibrium, all t: $g = \sqrt{\frac{1 + e^{-2c't}}{c'}}$

- Applied Field of the Form: $S(x) = c(t)x^{1/m}$
  - $\frac{dg}{dt} = \{g^{-1} + c(t)g^{1/m}\}^m$ if m = 1: $\frac{dg^2}{dt} = 2 + 2c(t)g^2$

Other Similarity Solutions

- Investigated:
  - Individual (constant) mobilities
  - Variable mobility dependent on position
- Mentioned
  - Individual Burgers vectors
  - Two-dimensional motions
  - Elastic boundaries
  - Other linear entities (electrostatic line charges, line currents, vortices, quantized flux lines, etc.)

Conclusions

- You can combine various scenarios investigated individually if you wish
- But the power law velocity-stress relationship is essential. Exponential laws do not have similarity motions, for example.
- Not everything has a similarity solution. Pile-ups, for example, don’t have a simple similarity solution like the ones above.