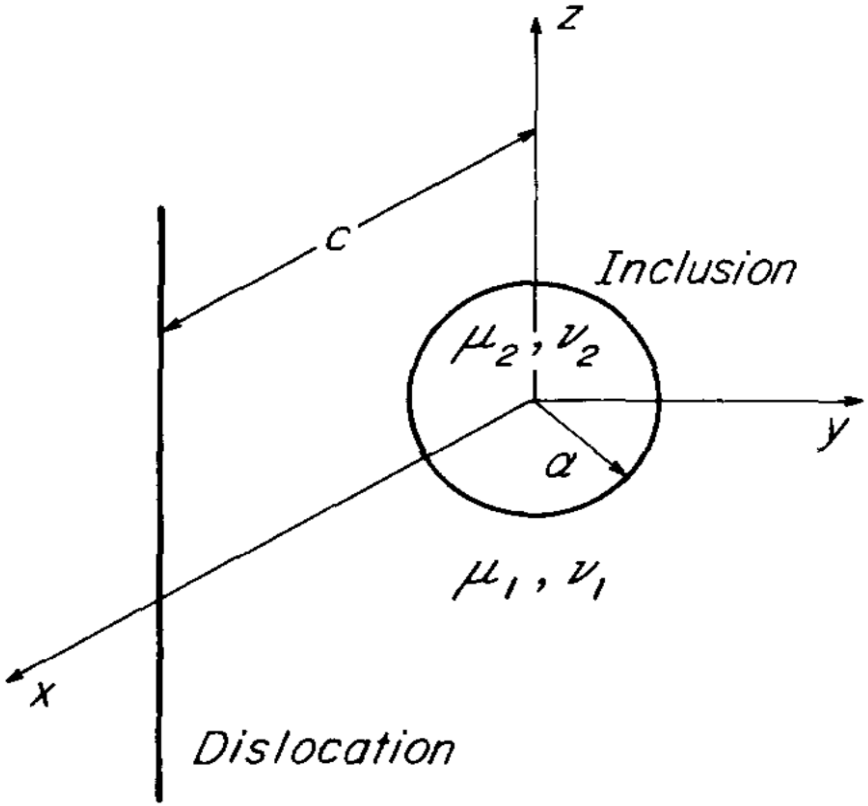
Long-Range Interaction between a Screw Dislocation and a Spherical Inclusion

Comninou and Dundurs (1972)



- Short paper discussing the long-range forces and interaction energy
- The stresses of the screw are:

$$\tau_{yz} = \frac{\mu_1 b_z}{2\pi} \frac{x-c}{y^2+(x-c)^2}, \quad \tau_{zx} = -\frac{\mu_1 b_z}{2\pi} \frac{y}{y^2+(x-c)^2}, \tag{1}$$

which at the origin of the coordinate system reduces to

$$\tau_{yz} = -\mu_1 b_z / 2\pi c \quad \text{and} \quad \tau_{zx} = 0. \tag{2}$$

- The long-range interaction is given by letting the inclusion disturb the uniform stress field in (2)

Inclusion Solution

- The displacements created by the inclusion in the matrix are:

$$u_x = -\frac{15\,T_{yz}}{2\,\mu_1} a^3\gamma \frac{xyz}{r^5}\left(1 - \frac{a^2}{r^2}\right), \qquad (3a)$$

$$u_y = \frac{T_{yz}}{2\,\mu_1}\left\{z - a^3\gamma \frac{z}{r^3}\left[5\left(1 - 2\nu_1 + \frac{3y^2}{r^2}\right) + \frac{3a^2}{r^3}\left(1 - \frac{5y^2}{r^2}\right)\right]\right\}, \qquad (3b)$$

$$u_z = \frac{T_{yz}}{2\,\mu_1}\left\{y - a^3\gamma \frac{y}{r^3}\left[5\left(1 - 2\nu_1 + \frac{3z^2}{r^2}\right) + \frac{3a^2}{r^2}\left(1 - \frac{5z^2}{r^2}\right)\right]\right\} \qquad (3c)$$

- In the inclusion:

$$u_x = 0,$$

$$u_y = \frac{T_{yz}}{2\,\mu_1}\,[1 - 2\gamma(4 - 5\nu_1)]z,$$

$$u_z = \frac{T_{yz}}{2\,\mu_1}\,[1 - 2\gamma(4 - 5\nu_1)]y$$

- Where Txy is the uniform shearing tractions applied to the matrix far away from the inclusion, and:

$$\gamma = \frac{\Gamma - 1}{2\Gamma(4 - 5\nu_1) + 7 - 5\nu_1}, \qquad \Gamma = \mu_2/\mu_1$$

Dislocation glide forces

- After some simplification we can obtain:

$$\sigma_{yz}^{d}(x,\ 0,\ z) = \frac{5\gamma b_z \mu_1 a^3}{2\pi c} \frac{1}{r^3}\left(1 - 2\nu_1 + 3\nu_1 \frac{z^2}{r^2}\right).$$

- Using the PK formula

$$F_x^{\infty} = b_z \sigma_{yz}^{d}(c,\ 0,\ z) = \frac{5\gamma b_z^2 \mu_1 a^3}{2\pi c} \frac{1}{r^3}\left(1 - 2\nu_1 + 3\nu_1 \frac{z^2}{r^2}\right),$$

- We see that the sign of F is determined by gamma, who's sign is determined by the ratio of shear moduli
  - The dislocation is repelled when the inclusion is stiffer than the matrix
- The distribution of the forces on the dislocation is highly dependent on the Poisson's ratio

Dislocation glide forces

- For Poisson's ratios between 0 and 0.25 the maximum force occurs at z=0 with a value of:

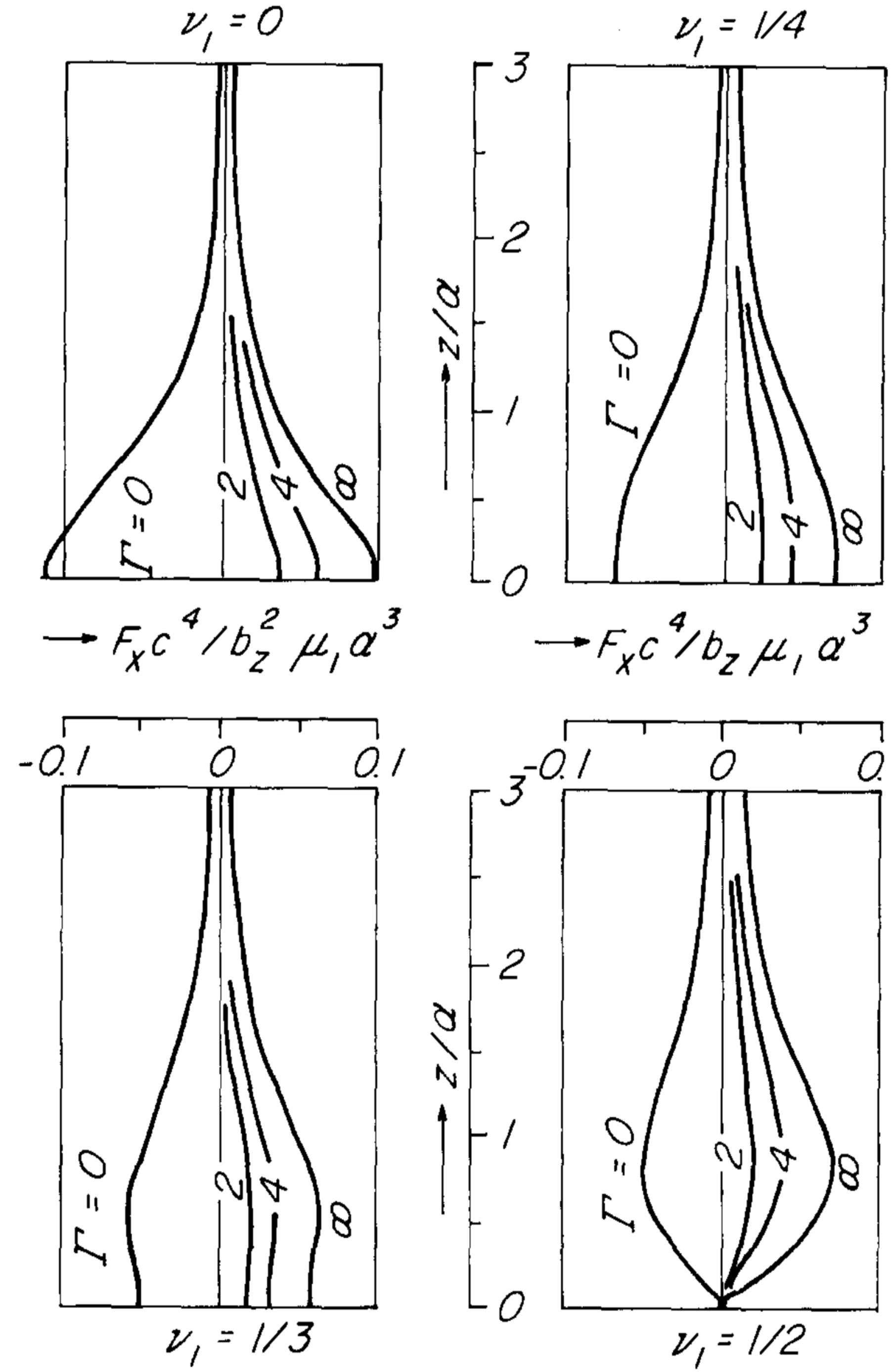
$$F_x^\infty = 5\gamma(1-2\nu_1)b_z^2\mu_1 a^3/2\pi c^4$$

- However, for nu>0.25 the max occurs at:

$$z = \pm\left[(4\nu_1-1)(1+\nu_1)^{-1}\right]^{1/2}a$$

- With a value of:

$$F_x^\infty = \gamma(1+\nu_1)^{5/2}b_z^2\mu_1 a^3\left[\pi(5\nu_1)^{3/2}c^4\right]^{-1}.$$



Minimum distance from inclusion

- For a cylindrical inclusion interaction energy can be computed exactly for c>a^3
  - A conservative estimate to give error within 10% is c >1.3a

- For a spherical inclusion a conservative estimate is c> 2.3a, to give an error within 10%
- The long-range always gives a lower bound on the energy (underestimates)

TABLE I. Comparison between long-range and exact energies and forces for a cylindrical inclusion.

c/a	E^{∞}/E	F_x^{∞}/F_x
2	0.866	0.750
3	0.943	0.889
4	0.968	0.938
5	0.979	0.960
6	0.986	0.972