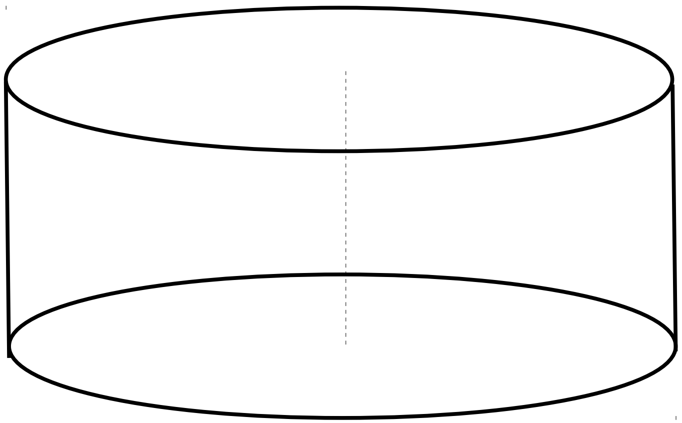
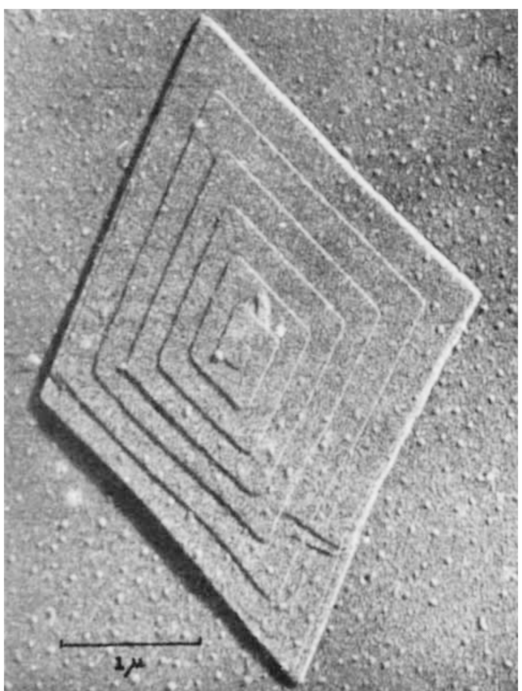
Dislocations in Thin Plates

Eshelby and Stroh (1951)





- Looks at the elastic fields of a screw dislocation normal to the surface of:
  - Semi-infinite half space
  - Semi-infinite plate
  - Disk of radius r_0 with or without a stress-free hole/core

- The work was motivated recent experimental images thin crystal growth images (Dawson and Vand 1951)

Half-space solution (z > 0)

- Infinite elastic fields of screw dislocation along z axis:

$$u_z = \frac{b}{2\pi}\,\theta \qquad \tau_{\theta z} = \frac{\mu b}{2\pi}\frac{1}{r}$$

- Eshelby states the image field to annul the traction on z=0

$$u_\theta = \frac{-b}{2\pi}\frac{r}{R+z} \qquad \tau_{\theta z} = \frac{-\mu b}{2\pi}\frac{r}{R(R+z)} \qquad \tau_{r\theta} = \frac{\mu b}{2\pi}\left(\frac{z^2}{R^3} + \frac{1}{R+z}\right)$$

$$R^2 = r^2 + z^2$$

- says this amounts to a distribution of force couples along the negative z-axis with density proportional to distance from origin

Infinite Plate (z = +/- d)

- Could be solved with an infinite series of half-space solutions, but they propose a different approach
- From the image solution we see that only components that don't vanish are u_θ $\tau_{\theta z}$ and $\tau_{r\theta}$
- Then: $$\tau_{\theta z} = \mu \frac{\partial u_\theta}{\partial z} \qquad \tau_{r\theta} = \mu \left(\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right)$$
- Equilibrium is: $$\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2} + \frac{\partial^2 u_\theta}{\partial z^2} = 0$$
- The solution is “easily” found to be:

$$u_\theta = \frac{-b}{2\pi} \int_0^\infty \frac{\sinh kz}{\cosh kd} \boldsymbol{J}_1(kr) \frac{dk}{k}$$

Infinite Plate (z = +/- d)

- The infinite plate solution can also be expressed as a series of modified Bessel function K1
- The finite disk solution (ri < r <ro) can be found by replacing K1 with a linear combination of K1 and I1

When $r_i=0$, $r_o=\infty$ the dislocation at the origin exerts a force

$$F=b'\int_{-d}^{d}\tau_{\theta z}\,dz=\frac{4bb'\mu}{\pi^2}\sum_{n\text{ odd}}n^{-1}K_1\left(\frac{n\pi r}{2d}\right)\quad .\ .\ .\ .$$

on another screw dislocation with Burgers vector b' distant r from it.

$$u_z=\frac{b}{2\pi}\theta,\qquad u_\theta=-\frac{b}{2\pi}\frac{z}{r},\qquad \tau_{r\theta}=\frac{\mu b}{\pi}\frac{z}{r^2},\qquad \tau_{\theta z}=0,$$

Interesting observations

- Shear stresses are no longer long range
- The energy of a screw dislocation in an infinite plate with its core excluded is finite
- Screw dislocations still attract, but no longer like Coulombic line charges
  - They now have a Yukawa potential
  - Reinforces what Prof. Barnett always says about Eshelby's grasp of electrostatics
- $\tau_{r\theta}$ does not vanish, screw dislocations will interact with parallel edge dislocations
  - Odd function, so only a force couple. No net force.

Edge dislocation

- Briefly mentioned, but not derived
- Requires higher-order bessel functions because of angular dependence of stresses
- The average stresses will still decay as 1/r
- Only buckling of the plate can lead to widespread relaxation
  - Describes how to demonstrate this with a piece of paper and the shape the free surface will take
- Buckling would only be energetically favorable in thin films. Solving would be non-linear and complicated
- Concludes with: “...in certain circumstances an edge dislocation will be able to relieve most of its stress by slight buckling... leading to a state of affairs similar to that of the screw.”